Monetary Economics

Chapter 3: Monetary-Policy Design

Olivier Loisel

ENSAE

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Goal of the chapter

- In Chapter 2, we assumed for simplicity that CB, at each date t,
  - directly controlled not only i_t, but also π_t and x_t,
  - observed the history of the exogenous shocks $(\overline{r}^n_{t-k}, u_{t-k})_{k\geq 0}$.

- In this chapter, we more relevantly assume that CB, at each date t,
  - directly controls only i_t,
  - may not observe the history of the exogenous shocks,

  and find that this affects the analysis in some important ways.

- The equation describing how CB sets i_t (as a function of some endogenous variables and/or exogenous shocks) is called "**interest-rate rule**."

- The main goal of the chapter is to address the question of what kind of interest-rate rule CB should follow, given its observation set.

Contribution of the chapter I

- We argue that the interest-rate rule should ensure **local-equilibrium determinacy**, i.e. be such that the log-linearized system made of the structural equations and this rule has a unique stationary solution.

- We state the **determinacy conditions** in a general framework, i.e. for a broad class of linear rational-expectations systems.

- We apply these general results to the log-linearized system made of
  - the structural equations of the basic NK model,
  - a simple interest-rate rule, like, e.g., Taylor's (1993) rule,

  and thus establish in particular the **Taylor principle**.

- We apply the latter results to **US monetary policy** between 1960 and 1996.

Contribution of the chapter II

- We also argue that the interest-rate rule should be such that this unique stationary solution coincides with the **optimal feasible path**, i.e. the path that maximizes RH's welfare subject to CB's observation-set constraint.

- We show that, in the basic NK model, for some reasonable observation sets of CB, there may exist no interest-rate rule
    - consistent with CB's observation set,
    - consistent with the optimal feasible path,
    - ensuring local-equilibrium determinacy,

  so that the **optimal feasible path may not be implementable**.

- Lastly, we explore the related but distinct issue of the **multiplicity of determinate projections** conditional on a given interest-rate path.

Outline of the chapter

1. Introduction
2. Determinacy conditions (Blanchard and Kahn, 1980)
3. Taylor principle (e.g., Woodford, 2003, C4)
4. Application to US monetary policy (Clarida, Galí and Gertler, 2000)
5. Implementability of optimal feasible MP (Loisel, 2020)
6. Multiplicity of determinate projections (Galí, 2011)

A class of linear rational-expectations systems

- In the first section of this chapter, we are interested in the conditions under which linear rational-expectations systems have a unique stationary solution.

- We consider the class of linear rational-expectations systems that can be written in Blanchard and Kahn's (1980) form:

$$\mathbb{E}_t \left\{ \boldsymbol{Z}_{t+1} \right\} = \boldsymbol{A}\boldsymbol{Z}_t + \boldsymbol{\xi}_t,$$

where

  - $\boldsymbol{Z}_t$ is a vector of endogenous variables set at date t or earlier,
  - $\boldsymbol{\xi}_t$ is a vector of exogenous disturbances realized at date t or earlier (often called "fundamental disturbances"),
  - $\boldsymbol{A}$ is a matrix with real-number elements.

Blanchard and Kahn's (1980) conditions

- Let m denote the number of **non-predetermined variables** of the system (loosely speaking, the number of degrees of freedom due to the presence of expected future variables).

- Let n denote the number of eigenvalues of $\boldsymbol{A}$ that are outside the unit circle.

- Blanchard and Kahn (1980) show that, provided that a certain rank condition is met (as is typically the case),
  - if $m < n$, then the system has no stationary solution,
  - if $m = n$, then the system has a unique stationary solution,
  - if $m > n$, then the system has an infinity of stationary solutions.

- Note that these conditions do not involve $\boldsymbol{\xi}_t$, so that they are the same for the deterministic system $\mathbb{E}_t \{\boldsymbol{Z}_{t+1}\} = \boldsymbol{A}\boldsymbol{Z}_t$.

A simple illustration

- Consider the following one-equation one-variable system:

$$\mathbb{E}_t \{z_{t+1}\} = a z_t + \xi_t,$$

where $a > 0$ and ξ_t is an i.i.d **fundamental shock** (i.e. shock that appears in the system).

- Blanchard and Kahn's (1980) conditions say that this system has
  - a unique stationary solution if $a > 1$,
  - an infinity of stationary solutions if $a < 1$.

- When $a > 1$, the unique stationary solution is $z_t = \frac{-\xi_t}{a}$.

- When $a < 1$, for any i.i.d. "**sunspot shock**" ζ_t (i.e. shock that does not appear in the system), $z_t = \frac{-\xi_t}{a} + \sum_{k=0}^{+\infty} a^k \zeta_{t-k}$ is a stationary solution.

Local-equilibrium determinacy

- When an economic system has an infinity of stat. solutions, sunspot shocks may make the economy more volatile, which typically decreases welfare.

- Therefore, an interest-rate rule should ensure **local-equilibrium determinacy**, i.e. be such that the log-linearized system made of the structural equations and this rule has a unique stationary solution.

- Bernanke and Woodford (1997) provide an easy-to-interpret example of multiple local equilibria.

- They consider an interest-rate rule prescribing to raise the short-term nominal interest rate i_t in response to a rise in the long-term nominal interest rate i_t^ℓ, interpreted rightly or wrongly as an "inflation scare."

- Then, markets' expectations of an increase in i_t will entail an increase in i_t^ℓ and therefore an increase in i_t that will validate these expectations.

Rules with an exogenous right-hand side I

- Consider any interest-rate rule setting the interest rate **exogenously** (i.e., as a function of exogenous shocks, with intercept r), and note it R_1.

- The deterministic version of the log-linearized system made of the IS equation, the Phillips curve and R_1 is

$$\begin{aligned} x_t &= \mathbb{E}_t\left\{x_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\} - r\right), \\ \pi_t &= \beta\mathbb{E}_t\left\{\pi_{t+1}\right\} + \kappa x_t, \\ i_t &= r, \end{aligned}$$

and can be rewritten as $i_t = r$ and

$$\left[\begin{array}{cc} \frac{1}{\sigma} & 1 \\ \beta & 0 \end{array}\right]\mathbb{E}_t\left\{\left[\begin{array}{c} \pi_{t+1} \\ x_{t+1} \end{array}\right]\right\} = \left[\begin{array}{cc} 0 & 1 \\ 1 & -\kappa \end{array}\right]\left[\begin{array}{c} \pi_t \\ x_t \end{array}\right].$$

Rules with an exogenous right-hand side II

- The latter system can be rewritten in Blanchard and Kahn's (1980) form $\mathbb{E}_t\left\{\boldsymbol{X}_{t+1}\right\} = \boldsymbol{A}_1\boldsymbol{X}_t$, where

$$\boldsymbol{X}_t \equiv \left[\begin{array}{c} \pi_t \\ x_t \end{array}\right] \text{ and } \boldsymbol{A}_1 \equiv \left[\begin{array}{cc} \frac{1}{\beta} & \frac{-\kappa}{\beta} \\ \frac{-1}{\beta\sigma} & 1+\frac{\kappa}{\beta\sigma} \end{array}\right].$$

- The eigenvalues of $\boldsymbol{A}_1$ are the real numbers

$$\begin{aligned} \delta &\equiv \frac{\left(1+\beta+\frac{\kappa}{\sigma}\right)-\sqrt{\left(1+\beta+\frac{\kappa}{\sigma}\right)^2-4\beta}}{2\beta} \in (0,1), \\ \delta' &\equiv \frac{\left(1+\beta+\frac{\kappa}{\sigma}\right)+\sqrt{\left(1+\beta+\frac{\kappa}{\sigma}\right)^2-4\beta}}{2\beta} > 1. \end{aligned}$$

Rules with an exogenous right-hand side III

- So, the system has two non-predetermined variables ($\mathbb{E}_t\{\pi_{t+1}\}$ and $\mathbb{E}_t\{x_{t+1}\}$), but only one eigenvalue outside the unit circle ($\delta' > 1$).
- Therefore, this system has multiple stationary solutions, i.e. there are **multiple local equilibria**.
- The result that setting the interest rate exogenously may lead to **indeterminacy** was first obtained by Sargent and Wallace (1975) in a non-micro-founded model.
- McCallum (1981) subsequently showed that interest-rate rules with an **endogenous right-hand side** could ensure determinacy, by making the interest rate react "out of equilibrium" to endogenous variables.

A Taylor rule I

- So consider, for instance, the following interest-rate rule, noted R_2:

$$i_t = r + \phi_\pi \pi_t + \phi_x x_t,$$

where $\phi_\pi \geq 0$ and $\phi_x \geq 0$, which is similar to **Taylor's** (1993) **rule**.

- Using this rule to replace i_t in the IS equation, we obtain the deterministic system $\mathbb{E}_t \left\{ \boldsymbol{X}_{t+1} \right\} = \boldsymbol{A}_2 \boldsymbol{X}_t$, where

$$\boldsymbol{A}_2 \equiv \left[\begin{array}{cc} \frac{1}{\beta} & \frac{-\kappa}{\beta} \\ \frac{\beta\phi_\pi - 1}{\beta\sigma} & 1 + \frac{\beta\phi_x + \kappa}{\beta\sigma} \end{array} \right],$$

so that R_2 ensures determinacy if and only if the two eigenvalues of $\boldsymbol{A}_2$ are outside the unit circle (since the system still has two non-predet. variables).

A Taylor rule II

- As shown by, e.g., Woodford (2003, C4), this happens if and only if

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1.$$

- The Phillips curve implies that a 1-unit permanent increase in the inflation rate leads to a $\frac{1-\beta}{\kappa}$-unit permanent increase in the output gap.

- So, the left-hand side of the **determinacy condition** above represents the permanent increase in the interest rate prescribed by R_2 in response to a 1-unit permanent increase in the inflation rate.

- Therefore, the determinacy condition corresponds to the **Taylor principle**: in the long term, the (nominal) interest rate should rise by more than the increase in the inflation rate.

A Taylor rule with inertia I

- Now consider the following interest-rate rule, noted R_3:

$$i_t = (1-\rho)\,r + \rho i_{t-1} + \phi_\pi \pi_t + \phi_x x_t,$$

where $\rho \geq 0$, $\phi_\pi \geq 0$ and $\phi_x \geq 0$, which includes R_2 as a special case.

- We then obtain the deterministic system $\mathbb{E}_t\left\{\boldsymbol{Y}_{t+1}\right\} = \boldsymbol{A}_3 \boldsymbol{Y}_t$, where

$$\boldsymbol{Y}_t \equiv \left[\begin{array}{c} \pi_t \\ x_t \\ i_{t-1} - r \end{array}\right] \quad \text{and} \quad \boldsymbol{A}_3 \equiv \left[\begin{array}{ccc} \frac{1}{\beta} & \frac{-\kappa}{\beta} & 0 \\ \frac{\beta\phi_\pi - 1}{\beta\sigma} & 1 + \frac{\beta\phi_x + \kappa}{\beta\sigma} & \frac{\rho}{\sigma} \\ \phi_\pi & \phi_x & \rho \end{array}\right],$$

so that R_3 ensures determinacy if and only if $\boldsymbol{A}_3$ has exactly two eigenvalues outside the unit circle (since the system still has two non-predet. variables).

A Taylor rule with inertia II

- As shown by, e.g., Woodford (2003, C4), this happens if and only if

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1-\rho.$$

- When $\rho < 1$, this **determinacy condition** corresponds once again to the **Taylor principle**: in the long term, the (nominal) interest rate should rise by more than the increase in the inflation rate.

- When $\rho \geq 1$, this determinacy condition is necessarily satisfied, and so is the Taylor principle since the prescribed increase in the interest rate is infinite.

- So, this determinacy condition corresponds to the Taylor principle no matter whether $\rho < 1$ or $\rho \geq 1$.

A forward-looking Taylor rule I

- Next, consider the following interest-rate rule, noted R_4:

$$i_t = r + \phi_\pi \mathbb{E}_t\{\pi_{t+1}\} + \phi_x x_t,$$

where $\phi_\pi \geq 0$ and $\phi_x \geq 0$.

- We then obtain the deterministic system $\mathbb{E}_t \left\{\boldsymbol{X}_{t+1}\right\} = \boldsymbol{A}_4 \boldsymbol{X}_t$, where

$$\boldsymbol{A}_4 \equiv \left[\begin{array}{cc} \frac{1}{\beta} & \frac{-\kappa}{\beta} \\ \frac{\phi_\pi - 1}{\beta\sigma} & 1 + \frac{\beta\phi_x - \kappa(\phi_\pi - 1)}{\beta\sigma} \end{array} \right],$$

so that R_4 ensures determinacy if and only if the two eigenvalues of $\boldsymbol{A}_4$ are outside the unit circle (since the system still has two non-predet. variables).

A forward-looking Taylor rule II

- As shown by, e.g., Woodford (2003, C4), this happens if and only if

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1$$

and

$$\phi_\pi < 1 + \frac{1+\beta}{\kappa}\left(\phi_x + \frac{2}{\sigma}\right).$$

- So, for this rule, the **Taylor principle** is necessary, but not sufficient for determinacy.

A forward-looking Taylor rule with inertia I

- Now turn to the following interest-rate rule, noted R_5:

$$i_t = (1-\rho)\, r + \rho i_{t-1} + \phi_\pi \mathbb{E}_t\{\pi_{t+1}\} + \phi_x x_t,$$

where $\rho \geq 0$, $\phi_\pi \geq 0$ and $\phi_x \geq 0$, which includes R_4 as a special case.

- We then obtain the deterministic system $\mathbb{E}_t\left\{\boldsymbol{Y}_{t+1}\right\} = \boldsymbol{A}_5 \boldsymbol{Y}_t$, where

$$\boldsymbol{A}_5 \equiv \begin{bmatrix} \frac{1}{\beta} & \frac{-\kappa}{\beta} & 0 \\ \frac{\phi_\pi - 1}{\beta\sigma} & 1 + \frac{\beta\phi_x - \kappa(\phi_\pi - 1)}{\beta\sigma} & \frac{\rho}{\sigma} \\ \frac{\phi_\pi}{\beta} & \phi_x - \frac{\kappa\phi_\pi}{\beta} & \rho \end{bmatrix},$$

so that R_5 ensures determinacy if and only if $\boldsymbol{A}_5$ has exactly two eigenvalues outside the unit circle (since the system still has two non-predet. variables).

A forward-looking Taylor rule with inertia II

- As shown by, e.g., Woodford (2003, C4), this happens only if

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1-\rho$$

and

$$\phi_\pi < 1+\rho+\frac{1+\beta}{\kappa}\left[\phi_x + \frac{2(1+\rho)}{\sigma}\right].$$

- So, for this rule too, the **Taylor principle** is necessary, but not sufficient for determinacy.

A Wicksellian rule I

- Lastly, consider the following interest-rate rule, noted R_6:

$$i_t = r + \phi_p p_t + \phi_x x_t,$$

where $\phi_p > 0$ and $\phi_x \geq 0$, which includes **Wicksell's** (1898) **rule** as a special case (namely the case in which $\phi_x = 0$).

- We then obtain the deterministic system $\mathbb{E}_t \{ \boldsymbol{Z}_{t+1} \} = \boldsymbol{A}_6 \boldsymbol{Z}_t$, where

$$\boldsymbol{Z}_t \equiv \begin{bmatrix} p_t \\ p_{t-1} \\ x_t \end{bmatrix} \text{ and } \boldsymbol{A}_6 \equiv \begin{bmatrix} \frac{1+\beta}{\beta} & \frac{-1}{\beta} & \frac{-\kappa}{\beta} \\ 1 & 0 & 0 \\ \frac{\beta\phi_p - 1}{\beta\sigma} & \frac{1}{\beta\sigma} & 1 + \frac{\beta\phi_x + \kappa}{\beta\sigma} \end{bmatrix},$$

so that R_6 ensures determinacy if and only if $\boldsymbol{A}_6$ has exactly two eigenvalues outside the unit circle (since the system still has two non-predet. variables).

A Wicksellian rule II

- As shown by, e.g., Woodford (2003, C4), this happens for any value of $\phi_p > 0$ and $\phi_x \geq 0$.
- This (absence of) **determinacy condition** corresponds once again to the **Taylor principle**.
- Indeed, any permanent increase in the inflation rate eventually leads to an infinite increase in the price level, and therefore an infinite increase in the interest rate.

An application to US monetary policy I

- We now turn to an **application** of the previous determinacy results.

- Clarida, Galí and Gertler (2000), henceforth CGG, argue that the observed decrease in **macroeconomic volatility** in the US between the 1960-1979 and 1979-1996 periods may be due to a change in MP in 1979.

- They estimate the following interest-rate rule on US data:

$$i_t = \rho(L) i_{t-1} + (1-\rho)\left[i^* + \beta\left(\mathbb{E}_t\left\{\pi_{t+k}\right\} - \pi^*\right) + \gamma \mathbb{E}_t\left\{x_{t+q}\right\}\right],$$

  where L is the lag operator ($Li_t \equiv i_{t-1}$), $\rho(L) \equiv \rho_1 + \rho_2 L + ... + \rho_n L^{n-1}$, $\rho \equiv \rho(1)$, and β does not denote the discount factor (in this section).

- The **Taylor principle** (neglecting the effect of a permanent increase in the inflation rate on the output gap) states that β should be higher than one.

Macroeconomic volatility in the US

TABLE I
AGGREGATE VOLATILITY INDICATORS

	Standard Deviation of:			
	Inflation		Output	
	Level	*hp*	*Gap*	*hp*
Pre-Volcker	2.77	1.48	2.71	1.83
Volcker-Greenspan	2.18	0.96	2.36	1.49
post–82	1.00	0.79	2.06	1.34

Source: Clarida, Galí and Gertler (2000).

An application to US monetary policy II

- CGG find that the estimated value of β is significantly
  - lower than one over the "**pre-Volcker**" period (1960-1979),
  - higher than one over the "**Volcker-Greenspan**" period (1979-1996).

- They conclude that MP
  - did not ensure determinacy during the pre-Volcker period,
  - did ensure determinacy during the Volcker-Greenspan period,

  which could contribute to explain the decrease in macroeconomic volatility.

- In fact, the condition considered ($\beta > 1$) may be neither necessary nor sufficient for determinacy under such a rule, but Lubik and Schorfheide (2004) re-do the exercise using the necessary and sufficient condition, for $n = 1$ and $k = q = 0$, and reach similar conclusions.

CGG's estimation method I

- Let $\Xi \equiv [\ \rho_1 \quad \cdots \quad \rho_n \quad \pi^* \quad \beta \quad \gamma\ ]'$ the vector of parameters to be estimated (i^* being calibrated, as π^* and i^* are not separately identifiable).

- How to estimate Ξ without using data about the private sector's expectations $\mathbb{E}_t\left\{\pi_{t+k}\right\}$ and $\mathbb{E}_t\left\{x_{t+q}\right\}$?

- The rational-expectations assumption implies that

$$\mathbb{E}\left\{\boldsymbol{Z}_t\left(\pi_{t+k}-\mathbb{E}_t\left\{\pi_{t+k}\right\}\right)\right\}=\mathbb{E}\left\{\boldsymbol{Z}_t\left(x_{t+q}-\mathbb{E}_t\left\{x_{t+q}\right\}\right)\right\}=0$$

  for any vector $\boldsymbol{Z}_t$ of variables (called "**instruments**") observed by the private sector when it forms its expectations at date t.

CGG's estimation method II

- We rewrite the interest-rate rule as

$$i_t = \rho(L)i_{t-1} + (1-\rho)\left[i^* + \beta\left(\pi_{t+k} - \pi^*\right) + \gamma x_{t+q}\right] \\ - (1-\rho)\left[\beta\left(\pi_{t+k} - \mathbb{E}_t\left\{\pi_{t+k}\right\}\right) + \gamma\left(x_{t+q} - \mathbb{E}_t\left\{x_{t+q}\right\}\right)\right].$$

- Multiplying by $\boldsymbol{Z}_t$ and applying $\mathbb{E}\{.\}$, we then get the **orthogonality condition**

$$\mathbb{E}\left\{\boldsymbol{Z}_t g_t(\Xi)\right\} = 0,$$

where $g_t(\Xi) \equiv i_t - \rho(L)i_{t-1} - (1-\rho)\left[i^* + \beta\left(\pi_{t+k} - \pi^*\right) + \gamma x_{t+q}\right]$.

- When $\dim(\boldsymbol{Z}_t) \geq \dim(\Xi)$, this provides the basis for the estimation of Ξ using Hansen's (1982) **generalized method of moments** (GMM).

GMM estimator

- Noting T the number of dates in the sample, we define
  - $\boldsymbol{m}(\Xi) \equiv \mathbb{E}\left\{\boldsymbol{Z}_t g_t(\Xi)\right\}$ the vector of **moments**,
  - $\widehat{\boldsymbol{m}}(\Xi) \equiv \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{Z}_t g_t(\Xi)$ the vector of **sample moments**.

- For any symmetric, positive and definite weight matrix $\boldsymbol{W}$, the **GMM estimator** is defined as

$$\widehat{\Xi}_{GMM} \equiv \arg\min_{\Xi} \left[\widehat{\boldsymbol{m}}(\Xi)' \boldsymbol{W} \widehat{\boldsymbol{m}}(\Xi)\right].$$

- For any weight matrix, the GMM estimator is **consistent** and **asymptotically normal**.

- When the weight matrix is the inverse of the variance-covariance matrix of the sample moments, the GMM estimator is also **asymptotically efficient**.

Hansen-Sargan test

- When $\dim(\boldsymbol{Z}_t) > \dim(\Xi)$, there are some **overidentifying restrictions** that can be tested with the **Hansen-Sargan test** (Hansen, 1982, Sargan, 1958).

- The null and alternative hypotheses of this test are
  - H_0: $\boldsymbol{m}(\Xi) = 0$,
  - H_1: $\boldsymbol{m}(\Xi) \neq 0$.

- The test statistic $J_T \equiv T\widehat{\boldsymbol{m}}(\widehat{\Xi}_{GMM})'\boldsymbol{W}_T\widehat{\boldsymbol{m}}(\widehat{\Xi}_{GMM})$, where $\boldsymbol{W}_T$ converges in probability towards the efficient weight matrix, is asymptotically
  - chi-squared with $\dim(\boldsymbol{Z}_t) - \dim(\Xi)$ degrees of freedom under H_0,
  - unbounded under H_1.

- In Tables II, III, IV and V, "p" denotes the p-value of J_T under H_0: for instance, H_0 is rejected at the 95% confidence level when $p < 0.05$.

CGG's benchmark and robustness analyses

- For the **benchmark estimation** (whose results are presented in Table II),
  - π is measured by the GDP-deflator inflation rate and x by the output gap constructed by the Congressional Budget Office,
  - the expectation horizons are $k = 1$ and $q = 1$,
  - the two periods considered are 1960Q1-1979Q2 and 1979Q3-1996Q4.

- The benchmark-estimation results are shown to be **robust** to the consideration of
  - alternative measures of π and x (in Table III),
  - alternative values of k and q (in Table IV),
  - different subperiods within each of the two periods (in Table V).

Benchmark-estimation results

TABLE II
BASELINE ESTIMATES

	π^*	β	γ	ρ	p
Pre-Volcker	4.24	0.83	0.27	0.68	0.834
	(1.09)	(0.07)	(0.08)	(0.05)	
Volcker-Greenspan	3.58	2.15	0.93	0.79	0.316
	(0.50)	(0.40)	(0.42)	(0.04)	

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.

Source: Clarida, Galí and Gertler (2000).

Robustness of the results I

TABLE III
ALTERNATIVE VARIABLES

	π^*	β	γ	ρ	p
Detrended output					
Pre-Volcker	4.17	0.75	0.29	0.67	0.801
	(0.68)	(0.07)	(0.08)	(0.05)	
Volcker-Greenspan	4.52	1.97	0.55	0.76	0.289
	(0.58)	(0.32)	(0.30)	(0.05)	
Unemployment rate					
Pre-Volcker	3.80	0.84	0.60	0.63	0.635
	(0.87)	(0.05)	(0.11)	(0.04)	
Volcker-Greenspan	4.42	2.01	0.56	0.73	0.308
	(0.44)	(0.28)	(0.41)	(0.05)	
CPI					
Pre-Volcker	4.56	0.68	0.28	0.65	0.431
	(0.53)	(0.06)	(0.08)	(0.05)	
Volcker-Greenspan	3.47	2.14	1.49	0.88	0.138
	(0.79)	(0.52)	(0.87)	(0.03)	

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation, output gap, the federal funds rate, the short-long spread, and commodity price inflation.

Source: Clarida, Galí and Gertler (2000).

Robustness of the results II

TABLE IV
ALTERNATIVE HORIZONS

	π^*	β	γ	ρ	p
$k = 4, q = 1$					
Pre-Volcker	3.58	0.86	0.34	0.73	0.835
	(1.42)	(0.05)	(0.08)	(0.04)	
Volcker-Greenspan	3.25	2.62	0.83	0.78	0.876
	(0.23)	(0.31)	(0.28)	(0.03)	
$k = 4, q = 2$					
Pre-Volcker	3.32	0.88	0.34	0.73	0.833
	(1.80)	(0.06)	(0.09)	(0.04)	
Volcker-Greenspan	3.21	2.73	0.92	0.78	0.886
	(0.21)	(0.34)	(0.31)	(0.03)	

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation, output gap, the federal funds rate, the short-long spread, and commodity price inflation.

Source: Clarida, Galí and Gertler (2000).

Robustness of the results III

TABLE V
SUBSAMPLE STABILITY

	π^*	β	γ	ρ	p
Martin					
(1,1)	5.16	0.86	0.14	0.77	0.524
	(1.72)	(0.08)	(0.16)	(0.06)	
(4,1)	7.15	0.92	0.06	0.72	0.719
	(5.55)	(0.08)	(0.07)	(0.05)	
Burns-Miller					
(1,1)	5.16	0.86	0.78	0.69	0.524
	(1.72)	(0.08)	(0.18)	(0.04)	
(4,1)	7.15	0.92	1.24	0.80	0.719
	(5.55)	(0.08)	(0.39)	(0.05)	
Volcker					
(1,1)	3.75	2.02	−0.02	0.63	0.612
	(0.28)	(0.23)	(0.15)	(0.04)	
(4,1)	2.45	2.38	0.68	0.74	0.804
	(0.47)	(0.35)	(0.30)	(0.04)	
Greenspan					
(1,1)	3.75	2.02	0.99	0.63	0.612
	(0.28)	(0.23)	(0.18)	(0.04)	
(4,1)	2.45	2.38	0.68	0.91	0.804
	(0.47)	(0.35)	(0.30)	(0.02)	
Post-82					
(1,1)	3.43	1.58	0.14	0.91	0.416
	(1.24)	(0.72)	(0.42)	(0.03)	
(4,1)	3.16	3.13	0.09	0.82	0.894
	(0.10)	(0.33)	(0.15)	(0.02)	

Standard errors are reported in parentheses. The set of instruments includes two lags of inflation, output gap, the federal funds rate, the short-long spread, and commodity price inflation, as well as the same variables with a multiplicative subperiod dummy.

Source: Clarida, Galí and Gertler (2000).

Structural equations and exogenous disturbances

- We now turn to the question of the **implementability of the optimal feasible path** in the basic NK model.

- Start with only two exogenous disturbances, affecting the discount factor and the elasticity of substitution between differentiated goods, both ARMA(1,1).

- The **structural equations** and **exogenous disturbances** are then

$$
\begin{aligned}
c_t &= \mathbb{E}_t\left\{c_{t+1}\right\} - \sigma^{-1}\left(i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\}\right) + \eta_t, \\
\pi_t &= \beta\mathbb{E}_t\left\{\pi_{t+1}\right\} + \kappa y_t + u_t, \\
y_t &= c_t, \\
n_t &= (1-\alpha)^{-1} y_t, \\
w_t &= \sigma c_t + \varphi n_t, \\
\eta_t &= \rho_\eta \eta_{t-1} + \varepsilon_t^\eta + \theta_\eta \varepsilon_{t-1}^\eta, \\
u_t &= \rho_u u_{t-1} + \varepsilon_t^u + \theta_u \varepsilon_{t-1}^u.
\end{aligned}
$$

CB's observation set and optimal feasible path

- Consider the following date-t **observation set** for CB: $O_t = \{c^{t-1}, \pi^{t-1}, y^{t-1}, n^{t-1}, w^{t-1}, i^{t-1}\}$, where, for any variable z, $z^t \equiv (z_{t-k})_{k\geq 0}$ denotes the history of z until date t included.

- The **optimal feasible path** (noted P) is the path minimizing, from Woodford's (1999) timeless perspective, $L_0 = \mathbb{E}_0\{\sum_{t=0}^{+\infty} \beta^t [(\pi_t)^2 + \lambda (y_t)^2]\}$, subject to the structural equations and the observation-set constraint.

- One can show that there exists an interest-rate rule (noted R) of type

$$i_t = \sum_{j=1}^{9} \left(f_j^{\pi} \pi_{t-j} + f_j^{y} y_{t-j} + g_j i_{t-j} \right),$$

such that any interest-rate rule consistent with O_t and P can be written in a form of type $\alpha(L)[i_t - \sum_{j=1}^{9}(f_j^{\pi}\pi_{t-j} + f_j^{y} y_{t-j} + g_j i_{t-j})] + \beta(L)[y_t - c_t] + \gamma(L)[n_t - (1-\alpha)^{-1} y_t] + \delta(L)[w_t - \sigma c_t - \varphi n_t] = 0$, where L denotes the lag operator, $[\alpha(X), \beta(X), \gamma(X), \delta(X)] \in \mathbb{R}[X]^4$, and $\alpha(0) \neq 0$.

(Non-)implementability of the optimal feasible path I

- Therefore, any rule consistent with O_t and P "robustly ensures determinacy" (i.e., ensures determinacy even when an exogenous MP shock is added to this rule) if only if R does.

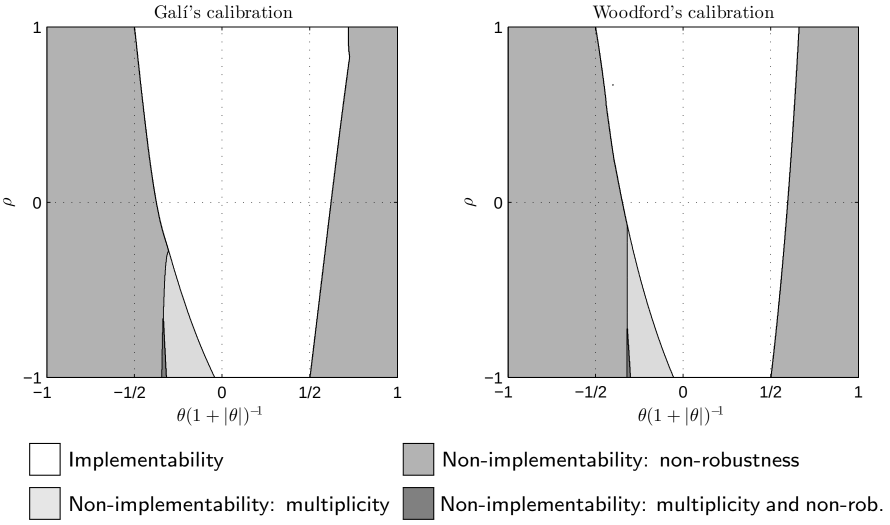
- Therefore, P is "implementable" (i.e., can be obtained as the robustly unique local equilibrium under a rule consistent with O_t) if and only if R robustly ensures determinacy.

- Consider two alternative calibrations for $(\beta, \sigma^{-1}, \kappa, \lambda)$:
  - $(0.99, 1.00, 0.125, 0.021)$ as in Galí (2015),
  - $(0.99, 6.25, 0.022, 0.003)$ as in Woodford (2003),

  and focus on values of $(\rho_\eta, \rho_u, \theta_\eta, \theta_u)$ such that $\rho_\eta = \rho_u \equiv \rho$ and $\theta_\eta = \theta_u \equiv \theta$ (note that θ is not a measure of price stickiness in this section).

- The next slide shows that P **is not implementable** for many values of ρ and θ (in particular for **news shocks**: $\theta \to +\infty$).

(Non-)implementability of the optimal feasible path II



Implications for the conduct of MP

- One key lesson of the NK literature is the importance for CBs to track some key unobserved exogenous **rates of interest** (Galí, 2015, Woodford, 2003).
- From a normative perspective, the most important of these rates of interest is the exogenous value taken by i_t on the optimal feasible path P.
- However, even when this value can be inferred in many alternative ways from O_t on P, there may be no way of setting i_t as a function of O_t that implements P as the robustly unique local equilibrium.
- In this case, any attempt to track this rate and implement P will result in
  - local-equilibrium multiplicity,
  - non-existence of a local equilibrium.

Robustness analysis

- Now introduce **three additional disturbances**, affecting government purchases, technology, and consumption utility or labor disutility.

- The Euler equation is left unchanged, the other structural equations become

$$\begin{aligned}
\pi_t &= \beta \mathbb{E}_t \{\pi_{t+1}\} + \kappa(y_t - \phi_g g_t - \phi_a a_t - \phi_\nu \nu_t) + u_t, \\
y_t &= (1-s)c_t + s g_t, \\
n_t &= (1-\alpha)^{-1} y_t - (1-\alpha)^{-1} a_t, \\
w_t &= \sigma c_t + \varphi n_t + \nu_t,
\end{aligned}$$

  and L_0 becomes $L_0 = \mathbb{E}_0 \{\sum_{t=0}^{+\infty} \beta^t [(\pi_t)^2 + \lambda(y_t - \phi_g g_t - \phi_a a_t - \phi_\nu \nu_t)^2]\}$.

- Assume that the additional disturbances follow ARMA processes and that $O_t = \{c^{t-1}, \pi^{t-1}, y^{t-1}, n^{t-1}, w^{t-1}, i^{t-1}, \varepsilon^{g,t-1}\}$.

- It is easy to show that we then get **exactly the same results** as previously.

Multiplicity of determinate projections I

- In the last section of this chapter, we address the related but distinct issue of **multiple determinate projections** (Galí, 2011).

- CBs typically do macroeconomic **projections**, i.e. macroeconomic **forecasts**, conditional on a given interest-rate path.

- In practice, there are **three main alternative assumptions** about the interest-rate path over the projection period:
  - the interest rate is constant,
  - the interest rate evolves according to markets' expectations,
  - the interest rate evolves according to CB's intentions.

- In the first two assumptions, the interest-rate path is given **exogenously**.

Multiplicity of determinate projections II

- If the interest-rate path were specified as exogenous in the projection exercise, then the projection would be **indeterminate**.

- So CBs usually consider an interest-rate rule ensuring determinacy and such that the path of the interest rate at the unique local equilibrium coincides with (or is close to) the exogenously given path.

- However, even though such projections are **determinate**, they are not **uniquely defined**.

- Indeed, for any exogenous interest-rate path, there exist several interest-rate rules ensuring determinacy and implementing
  - the same path for the interest rate,
  - different paths for the other endogenous variables,

  so that there is a **multiplicity of determinate projections**.

Three alternative interest-rate rules

- Consider an arbitrary exogenous path for the interest rate, noted $(i_t^*)_{t\in\mathbb{Z}}$.

- Consider three alternative interest-rate rules:
  - **Rule I**: $i_t = \varphi\pi_t + \nu_t$, where $\varphi > 1$ and ν_t is an exogenous term,
  - **Rule II**: $i_t = i_t^* - \gamma i_{t-1}^* + \gamma(\pi_t + \sigma\Delta x_t + \overline{r}_{t-1}^n)$, where $\gamma > 1$,
  - **Rule III**: $i_t = i_t^* - \gamma i_{t-1}^* + \gamma(\pi_t + r_{t-1})$, where $\gamma > 1$ and $r_t \equiv i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\}$ is the ex ante real short-term interest rate.

- All three rules ensure determinacy:
  - we have already shown that Rule I ensures determinacy,
  - it is easy to show, in a similar way, that so do Rules II and III.

- All three rules are such that $i_t = i_t^*$ at the unique local equilibrium:
  - in Rule I, ν_t is chosen such that this is indeed the case,
  - Rule II (combined with the IS equation) and Rule III imply $i_t - i_t^* = \frac{1}{\gamma}\mathbb{E}_t\left\{i_{t+1} - i_{t+1}^*\right\}$ and therefore $i_t = i_t^*$.

Three different local equilibria

- However, the three rules do not lead to the same unique local equilibrium, as they implement different paths for the inflation rate and the output gap.

- Therefore, the projection made at date t conditionally on $i_{t+k} = i^*_{t+k}$ for $k \geq 0$ is **not uniquely defined**.

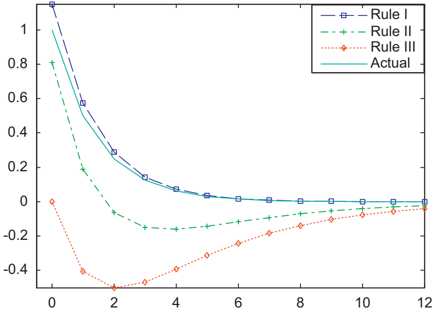
- This **multiplicity of determinate projections** is illustrated on the next three slides, for the constant-interest-rate (CIR) assumption,
  - first using the basic New Keynesian model (calibrated),
  - then using Smets and Wouters' (2007) DSGE model (estimated).

- As apparent on these slides, the difference between the projections can be quantitatively important.

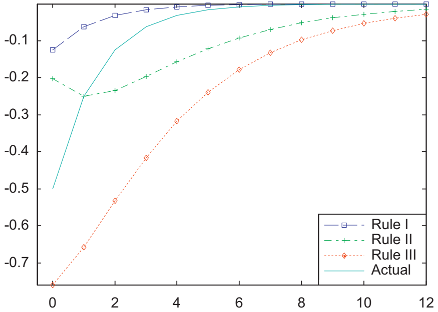
- On all three slides, the "actual rule" denotes the rule $i_t = \varphi \pi_t$ with $\varphi > 1$.

CIR-based responses to a cost-push shock in the NK mod.

Responses of the inflation rate



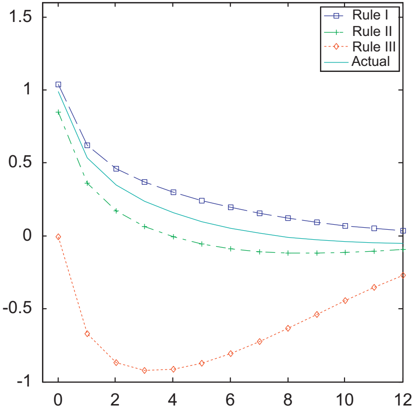
Responses of the output gap



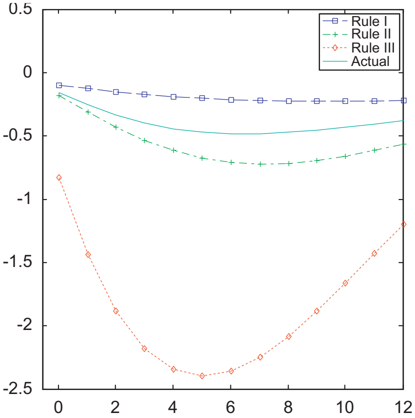
Source: Galí (2011).

CIR-based responses to a cost-push shock in SW's model

Responses of the inflation rate

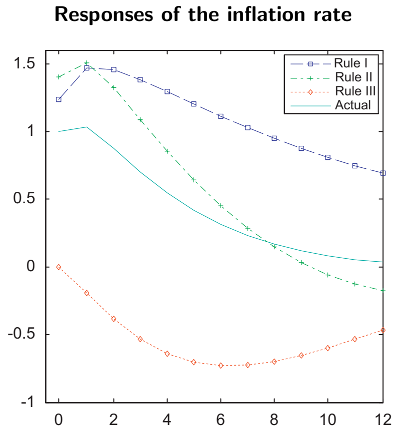


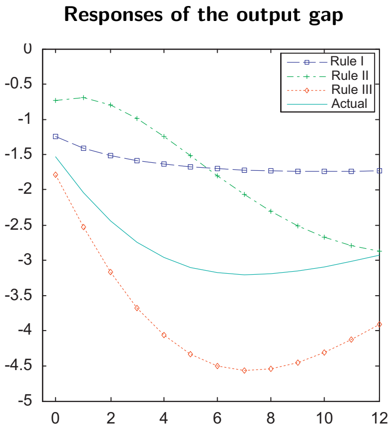
Responses of the output gap



Source: Galí (2011).

CIR-based responses to a technology shock in SW's model





Source: Galí (2011).